MINIMAL NON-SELFCENTRIC RADIALLY-MAXIMAL GRAPHS OF RADIUS 4

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Let G be a graph. By $E(G)$ we denote the edge set of G, and by $\overline{G}$ we denote the complement of G. The radius of G is denoted by $r(G)$ and the diameter of G is denoted by $d(G)$. We say that the graph G is **radially-maximal** if $r(G \cup e) < r(G)$ for every edge $e \in E(\overline{G})$.

Obviously, for every r there is a radially-maximal graph of radius r, as can be shown by complete graphs (in the case $r = 1$) and even cycles (in the case $r > 1$). However, both complete graphs and cycles are selfcentric graphs. Here we recall that a graph G is **selfcentric** if $r(G) = d(G)$. One may expect that a graph is radially-maximal if it is either a very dense graph or a balanced (highly symmetric) one. Therefore, it is interesting that for $r \geq 3$ there are non-selfcentric radially-maximal graphs of radius r which are planar (such graphs are neither symmetric nor planar). In fact, in [1] we have the following conjecture:

Conjecture 1 *Let G be a non-selfcentric radially-maximal graph with radius $r \geq 3$ on the minimum number of vertices. Then we have*
(a) *G has exactly $3r - 1$ vertices;*
(b) *G is planar;*
(c) *the maximum degree of G is 3 and the minimum degree of G is 1.*

In [1], Conjecture 1 was proved for the case $r = 3$. It was shown that there are just two non-selfcentric radially-maximal graphs of radius 3 on 8 vertices, namely the graphs depicted on Figure 1.

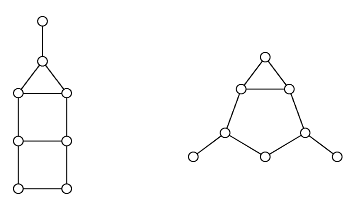

Figure 1

For higher values of r the conjecture is open. However, by an extensive computer search we found that there are exactly 8 graphs of radius 4 fulfilling all the conclusions of Conjecture 1. These graphs are depicted on Figure 2.

We remark that, even using a computer, it is not possible to examine all the graphs on 11 vertices. Therefore, our search went only through graphs on vertex set $\{v_1, v_2, \ldots, v_{11}\}$, in which the path $v_1, v_2, \ldots, v_6$ is geodesic. It means that $v_i v_{i+1}$ are edges for $1 \leq i \leq 5$ and there are no other edges inbetween the first six vertices. Moreover, the degree of v_1 is one and the maximum degree in G is 3.

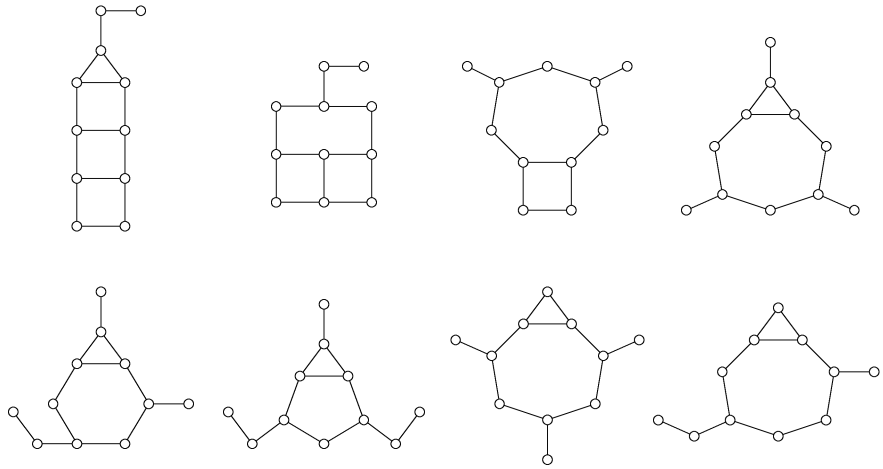

Figure 2

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References

[**1**] Gliviak, F., Knor, M., Šoltés, L'.: On radially maximal graphs, Australasian J. Comb. 9 (1994), pp. 275-284.

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